

## Answer on Question #80422 – Math – Calculus

### Question

Show that

$$\int_a^b \frac{\ln(x)}{x} dx = \frac{1}{2} \ln(ab) \ln\left(\frac{b}{a}\right)$$

### Solution

$$\begin{aligned} \int_a^b \frac{\ln(x)}{x} dx &= \int_a^b \ln(x) d \ln(x) = \frac{1}{2} \ln(x)^2 \Big|_a^b = \frac{1}{2} (\ln(b)^2 - \ln(a)^2) \\ &= \frac{1}{2} (\ln(b) - \ln(a)) (\ln(b) + \ln(a)) = \frac{1}{2} \ln\left(\frac{b}{a}\right) \ln(ab) \end{aligned}$$

### Answer:

The formula  $\int_a^b \frac{\ln(x)}{x} dx = \frac{1}{2} \ln(ab) \ln\left(\frac{b}{a}\right)$  was proved.