

Answer on Question #80411 – Math – Trigonometry

Question

$$\operatorname{Cosec}(\alpha) + \cot(\alpha) = 2\sqrt{3}$$

Check whether $\cos(\alpha) = 2/\sqrt{5}$

Solution

$$1/\sin(\alpha) + \cos(\alpha)/\sin(\alpha) = 2\sqrt{3} \quad \sin(\alpha) \neq 0$$

$$(1 + \cos(\alpha))/\sin(\alpha) = 2\sqrt{3}$$

$$((1 + \cos(\alpha))/\sin(\alpha))^2 = (2\sqrt{3})^2$$

$$(1 + 2\cos(\alpha) + \cos^2(\alpha))/\sin^2(\alpha) = 12$$

$$1 + 2\cos(\alpha) + \cos^2(\alpha) = 12(1 - \cos^2(\alpha))$$

$$13\cos^2(\alpha) + 2\cos(\alpha) - 11 = 0 \quad \cos(\alpha) = x$$

$$13x^2 + 2x - 11 = 0$$

$$x_{1,2} = (-2 \pm \sqrt{2^2 - 4 \cdot 13 \cdot (-11)}) / (2 \cdot 13) \quad x_1 = -1 \quad x_2 = 11/13$$

If $\cos(\alpha) = -1$, then in this case $\sin(\alpha) = 0$, hence $\operatorname{cosec}(\alpha)$ and $\cot(\alpha)$ do not exist, therefore $\cos(\alpha) = 11/13$.

Answer: $\cos(\alpha) = 11/13$