Answer on Question #80400 – Math – Analytic Geometry

Question

Obtain the equation of the parabola with focus (2,3) and directrix 3x − 4y + 9 = 0

Solution

Let (x,y) be a point on the parabola.

The distance between the point (x,y) and the directrix is the same distance from the point (x,y) to the focus:

$$\sqrt{(x-2)^2+(y-3)^2}=\frac{|3\cdot x-4\cdot y+9|}{\sqrt{3^2+(-4)^2}},$$

now by squaring both sides

$$\left(\sqrt{(x-2)^2+(y-3)^2}\right)^2=\left(\frac{|3\cdot x-4\cdot y+9|}{\sqrt{3^2+(-4)^2}}\right)^2;$$

$$(x-2)^2+(y-3)^2=\frac{(3\cdot x-4\cdot y+9)^2}{25}.$$

Simplifying

$$25\cdot\left(x^2-4\cdot x+4+y^2-6\cdot y+9\right)=\left((3\cdot x+9)-4\cdot y\right)^2;$$

$$25\cdot\left(x^2+y^2-4\cdot x-6\cdot y+13\right)=(3\cdot x+9)^2+2\cdot(-4\cdot y)\cdot(3\cdot x+9)+(-4\cdot y)^2;$$

$$25\cdot\left(x^2+y^2-4\cdot x-6\cdot y+13\right)=9\cdot x^2+54\cdot x+81-24\cdot x\cdot y-72\cdot y+16\cdot y^2;$$

$$25\cdot x^2+25\cdot y^2-100\cdot x-150\cdot y+325=9\cdot x^2+16\cdot y^2-24\cdot x\cdot y+54\cdot x-72\cdot y+81;$$

$$16\cdot x^2+9\cdot y^2+24\cdot x\cdot y-154\cdot x-78\cdot y+244=0.$$

Now we get the following equation

$$16\cdot x^2+9\cdot y^2+24\cdot x\cdot y-154\cdot x-78\cdot y+244=0.$$

Answer: $16\cdot x^2+9\cdot y^2+24\cdot x\cdot y-154\cdot x-78\cdot y+244=0.$