Answer on Question #80398 – Math – Calculus

Question

Find the angle of intersection between the curves x^2+2xy-y^2+2ax =0 and 3y^3-2a^2x-4a2y+a^3=0 at the point (a,-a).Please solve it is very urgent

Solution

First curve:

$x^2 + 2xy - y^2 + 2ax = 0$

Differentiate with respect to x:

$2x + 2xy' + 2y - 2yy' + 2a = 0,$

from which

$y' = \frac{x+y+a}{y-x}$

For $(x, y) = (a, -a)$:

$y' = \frac{a-a+a}{-a-a} = -\frac{1}{2}$

This is the tangent of angle between the first curve and x-axis, $\tan\theta_1 = -\frac{1}{2}$

Second curve:

$3y^3 - 2a^2x - 4a^2y + a^3 = 0$

Differentiate with respect to x:

$9y^2y' - 2a^2 - 4a^2y' = 0,$

from which

$y' = \frac{2a^2}{9y^2-4a^2}$

For $(x, y) = (a, -a)$:

$$y' = \frac{2a^2}{9a^2 - 4a^2} = \frac{2}{5}$$

This is the tangent of angle between the second curve and x-axis, $\tan\theta_2 = \frac{2}{5}$.

Then

$\tan(\theta_2 - \theta_1) = \frac{\tan\theta_2 - \tan\theta_1}{1+\tan\theta_2\tan\theta_1} = \frac{\frac{2}{5}+\frac{1}{2}}{1-\frac{2}{5}\cdot\frac{1}{2}} = \frac{9}{8}$

So the angle is $\theta = \arctan\frac{9}{8}$.