

Answer on Question #80393 – Math – Calculus

Question

Evaluate the integral $\int \sqrt{1 + \sqrt{x}} dx$.

Solution

$$\int \sqrt{1 + \sqrt{x}} dx,$$

we use the change of variables:

$$\sqrt{x} = t \quad \rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt \quad \rightarrow \quad dx = 2t dt.$$

It turns out

$$\int 2t\sqrt{1+t} dt.$$

For the subsequent solution, we use integration by parts: $\int u dv = uv - \int v du$.

$$u = t \quad \rightarrow \quad du = dt$$

$$dv = 2\sqrt{1+t} dt \quad \rightarrow \quad v = \frac{4}{3}(1+t)^{\frac{3}{2}}$$

$$\begin{aligned} \int 2t\sqrt{1+t} dt &= \frac{4}{3}t(1+t)^{\frac{3}{2}} - \int \frac{4}{3}(1+t)^{\frac{3}{2}} dt = \frac{4}{3}t(1+t)^{\frac{3}{2}} - \frac{8}{15}(1+t)^{\frac{5}{2}} + \text{constant} = \\ &= \frac{4}{15}(1+t)^{\frac{3}{2}}(3t-2) + \text{constant} \end{aligned}$$

Then

$$\int \sqrt{1 + \sqrt{x}} dx = \frac{4}{15}(\sqrt{x} + 1)^{\frac{3}{2}}(3\sqrt{x} - 2) + \text{constant}$$

Answer:

$$\frac{4}{15}(\sqrt{x} + 1)^{\frac{3}{2}}(3\sqrt{x} - 2) + \text{constant}$$