Answer on Question #80391 – Math – Calculus

Question

Say true or false. The tangent to the curve, $x^3 + 6y^2 + 5x = 0$ at $(-1, 1)$ is perpendicular to the y-axis.

Solution

If the tangent to the curve is perpendicular to the y-axis, then it is also parallel to the x-axis. The tangent at the point $(-1, 1)$ will be parallel to the x-axis if the following equality holds:

$$y'(-1) = 0$$

We express y in terms of x:

$$x^3 + 6y^2 + 5x = 0$$

$$y^2 = -\frac{x^3 + 5x}{6}$$

$$y = \sqrt{-\frac{x^3 + 5x}{6}}$$

We take only the square root with the positive sign because the y-coordinate of the (-1, 1) is positive.

We define the derivative of the function $y(x)$:

$$y'(x) = \left(\sqrt{-\frac{x^3 + 5x}{6}}\right)' = \frac{1}{2\sqrt{-\frac{x^3 + 5x}{6}}} \cdot \left(-\frac{x^3 + 5x}{6}\right)' = -\frac{3x^2 + 5}{2\sqrt{-6(x^3 + 5x)}}$$

$$y'(-1) = -\frac{2}{3}$$

As one can see, $y(-1) \neq 0$, then the curve at $(-1, 1)$ is not parallel to the x-axis and not perpendicular to the y-axis.

Answer:

The statement is false.