Answer on Question #80356 – Math – Financial Math

Question

B1 - A two year bond with a nominal rate of 3.5% per annum

these bonds have six monthly coupons and a face value of $2,000. Calculate their present values, Macauly durations and convexities using a YTM of 6% (YTM = 0.06).

Solution

Six coupons determine coupon payments at 3.5% of the face value of bonds of $ 2,000. Each coupon is $ 70. That is, CF for each coupon will have the following form

1 - $70;
2 - $70;
3 - $70;
4 - $70;
5 - $70;
6 - $2070 ($ 2000 + $ 70).

The first year - CF is 210 dollars, the second year at the end of payments - 2210 dollars. Present values are determined by the formula:

$$PV = \sum_{i=1}^{n} \frac{CF_i}{(1+r)^i} + \frac{N}{(1+r)^i}$$

N – nominal price of the bond;
r – YTM;
CF – cash flows.

Then

$$\frac{210}{1+0{,}06} + \frac{210}{(1+0{,}06)^2} + \frac{2000}{(1+0{,}06)^2} = 198{,}113 + 186{,}9 + 1779{,}99 = 2165$$

<u>*Present value*</u>= $ 2165.

Duration is a value measured by the duration of the time interval for which the nominal value of the security will be paid.

Duration is the elasticity of the bond price at an interest rate and therefore serves as a measure of the risk of a bond's price change when the interest rate changes.

It is determined by the formula:

$$D=\left[\sum_i \frac{CF_i}{(1+r)^i}\cdot t+\frac{nN}{(1+r)^n}\right]\cdot\frac{1}{P}$$

$$D=\left[\frac{1\cdot 210}{1+0.06}+\frac{2\cdot 210}{(1+0.06)^2}+\frac{2\cdot 2000}{(1+0.06)^2}\right]\cdot\frac{1}{2165}=1.9$$

The duration of the bonds will be *1.9 years*.

"Convexity" is a financial term and characterizes the dependence of the value of a bond on its yield. "Convexity" is directly related to duration and reflects the property of its change - the more the rate of change, the more the duration changes.

It is determined by the formula:

$$conv=\frac{1}{2}\frac{d^2PV}{dr^2}\cdot\frac{1}{PV}$$

$$conv=\frac{1}{2}\cdot\left[\sum_i \frac{CF_i}{(1+y)^i}\left(i^2+i\right)\right]\frac{1}{PV}$$

$$conv=\frac{1}{2}\left[\frac{2\cdot 210}{(1+0.06)^3}+\frac{4\cdot 2210}{(1+0.06)^4}\right]\cdot\frac{1}{2165}=\frac{1}{2}(352.6+7004.75)\cdot\frac{1}{2165}=1.69$$

Convexity is 1.69 years.

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https://www.finpipe.com/duration-macaulay-and-modified-duration-convexity/

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