Question

For the equation y = 5 + 4x^2 + 5x^3

(a) Find the equation for the linear approximation when x = 3.

(b) Find the equation for the quadratic approximation, also when x = 3.

Solution

(a) The tangent line to the function for x = 3 is the linear approximation:

$$L(x) = y(3) + y'(3)(x - 3)$$

$$y(3) = 5 + 4 \cdot 3^2 + 5 \cdot 3^3 = 176$$

$$y'(x) = 8x + 15x^2 \; \rightarrow \; y'(3) = 159$$

$$L(x) = 176 + 159(x - 3) = 159x - 301$$

(b) The quadratic approximation also uses the point x=3 to approximate nearby values, but uses a parabola instead of just a tangent line:

$$Q(x) = y(3) + y'(3)(x - 3) + \frac{1}{2}y''(3)(x - 3)^2 = L(x) + \frac{1}{2}y''(3)(x - 3)^2$$

$$y''(x) = 8 + 30x \; \rightarrow \; y''(3) = 98$$

$$Q(x) = 159x - 301 + \frac{1}{2}98(x - 3)^2 = 49x^2 - 135x + 140$$