Answer on Question #80306 – Math – Quantitative Methods

Question

For the equation y = 2 + 3 + 4x^2 + 5x^3

(a) Find the equation for the linear approximation when x = 3.

(b) Find the equation for the quadratic approximation, also when x = 3.

Solution

Maybe the function should be

$$y = 2 + 3x + 4x^2 + 5x^3 \quad (1)$$

because it makes no sense to write 2+3 instead of 5. I will do the question using the function (1).

We have

$f(3) = 2 + 3 \cdot 3 + 4 \cdot 3^2 + 5 \cdot 3^3 = 182$

Then

$f'(x) = 3 + 8x + 15x^2$,

$f'(3) = 3 + 8 \cdot 3 + 15 \cdot 3^2 = 162,$

$f''(x) = 8 + 30x$,

$f''(3) = 8 + 30 \cdot 3 = 98$.

(a) Linear approximation is

$y = f(a) + f'(a)(x - a)$

where $a = 3$.

Then we have

$y = 182 + 162(x - 3)$

or

$y = -304 + 162x.$

(b) Quadratic approximation is

$y = f(a) + f'(a)(x - a) + \frac{1}{2}f''(a)(x - a)^2$,

where $a = 3$.

Then we have

$$y = 182 + 162(x - 3) + \frac{98}{2}(x - 3)^2$$

or

$y = 137 - 132x + 49x^2$

Answer: a) $y = -304 + 162x$; **b)** $y = 137 - 132x + 49x^2$