

Answer on Question #80250 – Math – Quantitative Methods

Question

For the equation $y = 2 + 3x + 4x^2 + 5x^3$

(a) Find the equation for the linear approximation when $x = 3$.

Solution

$$L(x) = f(a) + f'(a)(x - a)$$

$$a = 3$$

$$f(3) = 2 + 9 + 36 + 135 = 182$$

$$f'(x) = 3 + 8x + 15x^2$$

$$f'(3) = 3 + 24 + 135 = 162$$

$$L(x) = 182 + 162 \cdot (x - 3)$$

$$L(x) = 182 + 162x - 486$$

$$L(x) = 162x - 304$$

Answer: $L(x) = 162x - 304$

Question

(b) Find the equation for the quadratic approximation, also when $x = 3$.

Solution

$$Q(x) = L(x) + \frac{f''(a)(x - a)^2}{2} = f(a) + f'(a)(x - a) + \frac{f''(a)(x - a)^2}{2}$$

$$f''(x) = 8 + 30x$$

$$f''(3) = 8 + 90 = 98$$

$$Q(x) = 162x - 304 + 49 \cdot (x - 3)^2$$

$$Q(x) = 162x - 304 + 49x^2 - 294x + 441$$

$$Q(x) = 49x^2 - 132x + 137$$

Answer: $Q(x) = 49x^2 - 132x + 137$