Question

i) The operation 5 defined by x5y = jln(xy)j where ln x is the natural logarithm.

ii) The operation 4 defined by x4y = x2+y3.

Also, for those operations which are binary operations, check whether they are associative and commutative

Solution

Associative property:

$$(a * b) * c = a * (b * c);$$

Commutative property:

$$a * b = b * a,$$

where * is binary operation.

i. $x5y = |\ln(xy)|$

Associative property:

$$(x5y)5z = |\ln(|\ln(xy)| \cdot z)| = |\ln(|\ln xy|) + \ln z|$$
$$x5(y5z) = |\ln(x \cdot |\ln yz|)| = |\ln x + \ln(|\ln yz|)|$$
$$(x5y)5z \neq x5(y5z)$$

— this operation is not associative.

Commutative property:

$$x5y = |\ln xy|$$

$$y5x = |\ln yx| = |\ln xy|.$$

$$x5y = y5x$$

— this operation is commutative.

ii. $x4y = x^2 + y^3$

Associative property:

$$(x4y)4z = (x^2 + y^3)^2 + z^3$$
$$x4(y4z) = x^2 + (y^2 + z^3)^3$$
$$(x4y)4z \neq x4(y4z)$$

— this operation is not associative.

Commutative property:

$$x4y = x^2 + y^3$$

$$y4x = y^2 + x^3$$

$$x4y \neq y4x$$

— this operation is not commutative.