Answer on Question #79918 – Math – Linear Algebra

Question

Which of the following statements are true and which are false? Justify your answer with a short proof or a counterexample.

i) R2 has infinitely many non-zero, proper vector subspaces.

ii) If T : V !W is a one-one linear transformation between two finite dimensional vector spaces V andW then T is invertible.

iii) If Ak = 0 for a square matrix A, then all the eigenvalues of A are zero.

iv) Every unitary operator is invertible.

v) Every system of homogeneous linear equations has a non-zero solution

Solution

i) Yes, amount of different 1-dimensional subspaces L={(x,y)} with any x and y is infinite. We need just find infinite numbers of uncollinear vectors. For exampls, it will be (sin t, cos t) for t from 0 to pi. There are infinite uncollinear vectors, and it causes infinite amount of one-dimensional subspaces. We do not consider 2-dimensional subspaces here, two-dimensional space is single in R2 and it is the whole space R2 because if subspace of space have the same dimension as this space, this subspace and this space are equal.

ii) Yes,firstly V and W have same dimension.Obviously, then ker T ={0}, and this is sufficient for existence of T^-1. T is one-one linear transformation, so T move one basis to another one. If the amount of vectors in the bases are not same, there exists at least one vector which will be linearly dependent on the others, but our transformation is one-one. The T^-1 is just a transformation, which all vectors of a basis in the second space moves in the opposite direction.

iii) Yes, it is one of properties of nilpotent matrix, a proof can be found at https://yutsumura.com/nilpotent-matrix-and-eigenvalues-of-the-matrix/ .

iv) Yes because detA=+-1 and it is not equal to 0.

v) No. Indeed, the system
$x+y=0$
$x+2y=0$
has only the zero solution ($x=0$, $y=0$).