Question

Check whether the following system of equations has a solution.

$4x+2y+8z+6w = 3$

$2x+2y+2z+2w = 1$

$x+3z+2w = 3$

Solution

Find a rank of the coefficient matrix A:

$$A = \begin{pmatrix} 4 & 2 & 8 & 6 \\ 2 & 2 & 2 & 2 \\ 1 & 0 & 3 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0.5 & 2 & 1.5 \\ 2 & 2 & 2 & 2 \\ 1 & 0 & 3 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0.5 & 2 & 1.5 \\ 0 & 1 & -2 & -1 \\ 0 & -0.5 & 1 & 0.5 \end{pmatrix} \sim \begin{pmatrix} 1 & 0.5 & 2 & 1.5 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Rank(A) = 2.

Write an augmented matrix of the system of the equations and find her rank:

$$B = \begin{pmatrix} 4 & 2 & 8 & 6 & 3 \\ 2 & 2 & 2 & 2 & 1 \\ 1 & 0 & 3 & 2 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0.5 & 2 & 1.5 & 0.75 \\ 2 & 2 & 2 & 2 & 1 \\ 1 & 0 & 3 & 2 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0.5 & 2 & 1.5 & 0.75 \\ 0 & 1 & -2 & -1 & -0.5 \\ 0 & -0.5 & 1 & 0.5 & 2.25 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0.5 & 2 & 1.5 & 0.75 \\ 0 & 1 & -2 & -1 & -0.5 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0.5 & 2 & 1.5 & 0.75 \\ 0 & 1 & -2 & -1 & -0.5 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Rank(B) = 3.

Rank(A) ≠ Rank(B). By Kronecker-Capelli theorem, the given system of linear equations is incompatible, the system has no solution.

Answer: the system has no solution.