

Answer on Question #79908 – Math – Linear Algebra

Question

Let $(x_1; x_2; x_3)$ and $(y_1; y_2; y_3)$ represent the coordinates with respect to the bases

$$B_1 = \{(1; 0; 0), (0; 1; 0), (0; 0; 1)\}, B_2 = \{(1; 0; 0), (0; 1; 2), (0; 2; 1)\}.$$

If

$$Q(X) = x_1^2 + 2x_1x_2 + 2x_2x_3 + x_2^2 + x_3^2,$$

find the representation of Q in terms of $(y_1; y_2; y_3)$.

Solution

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$P = P_{B_1 \rightarrow B_2} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array} \right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -3 & 0 & -2 & 1 \end{array} \right] =$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 2/3 \\ 0 & 0 & 1 & 0 & 2/3 & -1/3 \end{array} \right]$$

$$A_2 = P^T A P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/3 & 2/3 \\ 0 & 2/3 & -1/3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/3 & 2/3 \\ 0 & 2/3 & -1/3 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/3 & 2/3 \\ 0 & 2/3 & -1/3 \end{bmatrix} \begin{bmatrix} 1 & -1/3 & 2/3 \\ 1 & 1/3 & 1/3 \\ 0 & 1/3 & 1/3 \end{bmatrix} = \begin{bmatrix} 1 & -1/3 & 2/3 \\ -1/3 & 1/9 & 1/9 \\ 2/3 & 1/9 & 1/9 \end{bmatrix}$$

Answer:

$$Q(Y) = y_1^2 + 1/9 y_2^2 + 1/9 y_3^2 - 2/3 y_1 y_2 + 4/3 y_1 y_3 + 2/9 y_2 y_3.$$