Question

Find the orthogonal canonical reduction of the quadratic form $x^2 + y^2 + z^2$.

Solution

A quadratic form in 3 variables $Q(x, y, z)$ is a function of the form $Q(x, y, z) = q_{xx}x^2 + 2 \cdot q_{xy}xy + 2 \cdot q_{xz}xz + q_{yy}y^2 + 2 \cdot q_{yz}xy + q_{zz}z^2$. It is known that by changing the system of coordinates $(x, y, z) \rightarrow (x', y', z')$ in some way, any quadratic form can be transformed to a canonical form $Q(x, y, z) = D(x', y', z') = d_{x'x'}x'^2 + d_{z'z'}y'^2 + d_{z'z'}z'^2$. The reduction is called orthogonal if the transformation is defined by an orthogonal matrix.
In the case of this problem, the quadratic form is already a linear combination of the squares of the variables. And the trivial transformation is defined by an orthogonal matrix, i.e. the identity matrix.

Answer:

The quadratic form $x^2 + y^2 + z^2$ is a orthogonal canonical form.