Question

Find domain and range of the function $f(x) = \dfrac{1}{\left|x^2 - 4\right|}$.

Solution

Solve to find the value of x that makes the expression $f(x) = \dfrac{1}{\left|x^2 - 4\right|}$ undefined. So,

$$\left|x^2 - 4\right| = 0,$$

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$$x^2 = 4,$$

$$x = \pm\sqrt{4},$$

$$x = \pm 2.$$

The domain is all values of x that make the expression defined. Therefore, the <u>domain</u> is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

The range is the set of all valid y values. Since $\left|x^2 - 4\right| \geq 0$, $y \geq 0$. Note that $y \neq 0$. Consider any $y > 0$. Then

$$y = \frac{1}{\left|x^2 - 4\right|}, \quad \left|x^2 - 4\right| = \frac{1}{y},$$

$$x^2 - 4 = \pm\frac{1}{y}, \quad x^2 = 4 \pm\frac{1}{y}.$$

So, for $x^2 = 4 + \dfrac{1}{y}$ we have that $x = \sqrt{4 + \dfrac{1}{y}}$.

Hence, the range is $(0, \infty)$.

Answer: the domain is $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$; the range is $(0,\infty)$.