# Answer on Question #79869 – Math – Linear Algebra

## Question

Check whether the matrices A and B are diagonalizable. Diagonalize those matrices which are diagonalizable. (11)

**i)**

$$A = \begin{pmatrix} -2 & -5 & -1 \\ 3 & 6 & 1 \\ -2 & -3 & 1 \end{pmatrix}$$

**ii)**

$$B = \begin{pmatrix} -1 & -3 & 0 \\ 2 & 4 & 0 \\ -1 & -1 & 2 \end{pmatrix}$$

## Solution

**i)** $A = \begin{pmatrix} -2 & -5 & -1 \\ 3 & 6 & 1 \\ -2 & -3 & 1 \end{pmatrix}$

Characteristic equation:

$$\begin{vmatrix} -2-\lambda & -5 & -1 \\ 3 & 6-\lambda & 1 \\ -2 & -3 & 1-\lambda \end{vmatrix} = (-2-\lambda)(6-7\lambda+\lambda^2+3)+5(3-3\lambda+2)-(-9+12-2\lambda) =$$

$$= (-2-\lambda)(\lambda^2-7\lambda+9)+25-15\lambda-3+2\lambda = -\lambda^3-2\lambda^2+7\lambda^2+14\lambda-9\lambda-18-13\lambda+22 =$$

$$= -(\lambda^3-5\lambda^2+8\lambda-4) = -(\lambda-1)(\lambda-2)^2$$

Eigenvalues are $\lambda_1 = 1, \lambda_2 = 2$. There is one eigenvector corresponding to $\lambda_1$ . There can be one or two eigenvectors corresponding to $\lambda_2$. We need to define which of these cases holds now.

$$\begin{pmatrix} -4 & -5 & -1 \\ 3 & 4 & 1 \\ -2 & -3 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{cases} -4x_1-5x_2-x_3=0 \\ 3x_1+4x_2+x_3=0 \\ -2x_1-3x_2-x_3=0 \end{cases}$$

$$\begin{cases} x_3=-4x_1-5x_2 \\ -x_1-x_2=0 \\ 2x_1+2x_2=0 \end{cases}$$

$$\begin{cases} x_2=-x_1 \\ x_3=-4x_1-5x_2 \end{cases}$$

$(x_1, x_2, x_3) = (t, -t, t) = t(1, -1, 1)$ is linear space of dimension 1 on the vector (1, -1, 1). Thus, $(1, -1, 1)$ is the one eigenvector corresponding to $\lambda_2 = 2$. So there will be two eigenvectors in total.

Since the number 2 of eigenvectors is less than dimension 3 of the matrix, then the matrix is not diagonalizable.

**ii)** $B = \begin{pmatrix} -1 & -3 & 0 \\ 2 & 4 & 0 \\ -1 & -1 & 2 \end{pmatrix}$

Characteristic equation:

$$\begin{vmatrix} -1-\lambda & -3 & 0 \\ 2 & 4-\lambda & 0 \\ -1 & -1 & 2-\lambda \end{vmatrix} = (-1-\lambda)(4-\lambda)(2-\lambda) + 3\cdot 2(2-\lambda) = -\lambda^3 + 5\lambda^2 - 2\lambda - 8 + 12 - 6\lambda =$$

$$= -(\lambda^3 - 5\lambda^2 + 8\lambda - 4) = -(\lambda-1)(\lambda-2)^2$$

Eigenvalues are $\lambda_1 = 1, \lambda_2 = 2$. There is one eigenvector corresponding to $\lambda_1 = 1$ . There can be one or two eigenvectors corresponding to $\lambda_2 = 2$. We need to define which of these cases holds now.

$$\begin{pmatrix} -3 & -3 & 0 \\ 2 & 2 & 0 \\ -1 & -1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{cases} -3x_1 - 3x_2 = 0 \\ 2x_1 + 2x_2 = 0 \\ -x_1 - x_2 = 0 \end{cases}$$

$$\{x_2 = -x_1$$

$(x_1, x_2, x_3) = (t, -t, z) = t(1,-1,0) + z(0,0,1)$ is linear space of dimension 2 on vectors (1,-1,0) and (0,0,1).

The vectors $(1,-1,0), (0,0,1)$ are two eigenvectors corresponding to $\lambda_2 = 2$. So there will be three eigenvectors in total, that's why the matrix is diagonalizable.

Find the remaining eigenvector corresponding to $\lambda_1 = 1$:

$$\begin{pmatrix} -2 & -3 & 0 \\ 2 & 3 & 0 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{cases} -2x_1 - 3x_2 = 0 \\ 2x_1 + 3x_2 = 0 \\ -x_1 - x_2 + x_3 = 0 \end{cases}$$

$$\begin{cases} x_2 = -\frac{2}{3}x_1 \\ x_3 = x_1 + x_2 \end{cases}$$

$$\begin{cases} x_2 = -\frac{2}{3}x_1 \\ x_3 = \frac{1}{3}x_1 \end{cases}$$

The solution, the eigenvector is $(3,-2,1)$.

Diagonalization of the matrix *B*.

$$B = \begin{pmatrix} 3 & 1 & 0 \\ -2 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 & 0 \\ -2 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{-1}$$