Answer on Question #79754 - Math - Discrete Mathematics
August 13, 2018

Question. Let $A = \{1; 2; 3; 4; 5; 6; 7; 8; 9; 10\}$ and $B = \{1; 2; 3; 4\}$. Let R be the relation on $\mathcal{P}(A)$ defined by:

For any X, Y element in $\mathcal{P}(A)$, XRY if and only if $X - B = Y - B$. How many equivalence classes are there? Explain.

Answer. Define a function f on $\mathcal{P}(A)$ by $f(X) = X - B$. Then XRY if and only if $f(X) = f(Y)$.

We will show that there is a one-to-one correspondence from R-equivalence classes to the range of f denoted by range(f). For every equivalence class with a representative X, let $f(X)$ correspond to this equivalence class. As every equivalence class has at least one representative, at least one element of range(f) corresponds to every equivalence class. If X and Y are representatives of the same equivalence class, then XRY, $f(X) = f(Y)$, hence at most one element of range(f) corresponds to this equivalence class.

The range of f is $\mathcal{P}(A - B)$, as we will show.

- Assume that $X \subseteq A - B$. $X - B \subseteq X$. Every $x \in X$ does not belong to B (because $X \subseteq A - B$), so $x \in X - B$. Hence $f(X) = X - B = X$, and $X \in$ range(f).
- Assume that $X \in$ range(f). Then there is $Y \subseteq A$ such that $f(Y) = X$. Hence $X = Y - B$. Every element of X belongs to Y, hence it belongs to A, and does not belong to B, so $X \subseteq A - B$.

$A - B = \{5; 6; 7; 8; 9; 10\}$. The set $A - B$ has exactly 6 members. It follows that $\mathcal{P}(A - B)$ has exactly $2^6 = 64$ elements. Therefore, there are exactly 64 R-equivalence classes.