Question

$$1. \quad \frac{dx}{dt} = 2x, \frac{dy}{dt} = 4y$$

Solution

Matrix of the system:

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$$

Solve the characteristic equation:

$$det(A - \lambda E) = \begin{vmatrix} 2 - \lambda & 0 \\ 0 & 4 - \lambda \end{vmatrix} = 0$$
$$(2 - \lambda)(4 - \lambda) = 0$$
$$\lambda_1 = 2 , \ \lambda_2 = 4$$

Find eigenvectors of the matrix:

a) $\lambda_1=2$

$$\begin{pmatrix} 2-2 & 0 \\ 0 & 4-2 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}$$
$$\begin{cases} \alpha_1 = 2C_1 \\ \alpha_2 = 0 \end{cases} \Longrightarrow \underset{A_1}{\longrightarrow} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

b) $\lambda_2=4$

$$\begin{pmatrix} 2-4 & 0 \\ 0 & 4-4 \end{pmatrix} \sim \begin{pmatrix} -2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{cases} \alpha_1 = 0\\ \alpha_2 = -2C_1 \end{cases} \Longrightarrow \underset{A_2}{\longrightarrow} = \begin{pmatrix} 0\\ -2 \end{pmatrix}$$
$$\begin{pmatrix} x\\ y \end{pmatrix} = C_1 \begin{pmatrix} 2\\ 0 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 0\\ -2 \end{pmatrix} e^{4t}$$

Answer:
$$\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 0 \\ -2 \end{pmatrix} e^{4t}.$$

Question

$$2. \quad \frac{dx}{dt} = 2x, \frac{dy}{dt} = 2y$$

Solution

Matrix of the system:

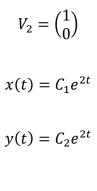
$$A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Solve the characteristic equation:

$$det(A - \lambda E) = \begin{vmatrix} 2 - \lambda & 0 \\ 0 & 2 - \lambda \end{vmatrix} = 0$$
$$(2 - \lambda)^2 = 0$$
$$\lambda = 2$$

Find eigenvectors of the matrix:

$$\begin{pmatrix} 2-2 & 0 \\ 0 & 2-2 \end{pmatrix} \begin{pmatrix} V_{11} \\ V_{21} \end{pmatrix} = 0$$
$$\begin{cases} 0 \cdot V_{11} + 0 \cdot V_{21} = 0 \\ 0 \cdot V_{11} + 0 \cdot V_{21} = 0 \end{cases}$$
$$V_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Answer:

 $x(t) = C_1 e^{2t}$

 $y(t) = C_2 e^{2t}$

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