Answer on Question #79727 – Math – Differential Equations

Question

1. $\frac{dx}{dt} = 2x, \frac{dy}{dt} = 4y$

Solution

Matrix of the system:

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$$

Solve the characteristic equation:

$$\det(A - \lambda E) = \begin{vmatrix} 2-\lambda & 0 \\ 0 & 4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(4-\lambda) = 0$$

$$\lambda_1 = 2\ ,\ \lambda_2 = 4$$

Find eigenvectors of the matrix:

a) $\lambda_1 = 2$

$$\begin{pmatrix} 2-2 & 0 \\ 0 & 4-2 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}$$

$$\begin{cases} \alpha_1 = 2C_1 \\ \alpha_2 = 0 \end{cases} => \underset{A_1}{\rightarrow} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

b) $\lambda_2 = 4$

$$\begin{pmatrix} 2-4 & 0 \\ 0 & 4-4 \end{pmatrix} \sim \begin{pmatrix} -2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{cases} \alpha_1 = 0 \\ \alpha_2 = -2C_1 \end{cases} => \underset{A_2}{\rightarrow} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 0 \\ -2 \end{pmatrix} e^{4t}$$

Answer: $\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 0 \\ -2 \end{pmatrix} e^{4t}$.

Question

2. $\frac{dx}{dt} = 2x, \frac{dy}{dt} = 2y$

Solution

Matrix of the system:

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Solve the characteristic equation:

$$\det(A - \lambda E) = \begin{vmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)^2 = 0$$

$$\lambda = 2$$

Find eigenvectors of the matrix:

$$\begin{pmatrix} 2-2 & 0 \\ 0 & 2-2 \end{pmatrix} \begin{pmatrix} V_{11} \\ V_{21} \end{pmatrix} = 0$$

$$\begin{cases} 0 \cdot V_{11} + 0 \cdot V_{21} = 0 \\ 0 \cdot V_{11} + 0 \cdot V_{21} = 0 \end{cases}$$

$$V_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$V_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$x(t) = C_1 e^{2t}$$

$$y(t) = C_2 e^{2t}$$

Answer:

$$x(t) = C_1 e^{2t}$$

$$y(t) = C_2 e^{2t}$$