Answer on Question #79725 – Math – Differential Equation

Question

Differential equations in the form of $F(x, y, p), F(y/x, p)$ or differential equations containing $p(p = dy/dx)$

1.

$$4yp^2 + 2xp - y = 0$$

Solution

$$y(4p^2 - 1) + 2xp = 0$$

$$y = \frac{2xp}{1 - 4p^2}$$

$$\frac{dy}{dx} = p = \frac{2p}{1 - 4p^2} + 2x\frac{p'(x)(1 - 4p^2) + 8pp'(x)}{(1 - 4p^2)^2}$$

$$p - \frac{2p}{1 - 4p^2} = 2 \cdot \frac{1 - 4p^2 + 8p}{(1 - 4p^2)^2} xp'(x)$$

$$\frac{p - 4p^3 - 2p}{1 - 4p^2} = 2 \cdot \frac{1 - 4p^2 + 8p}{(1 - 4p^2)^2} xp'(x)$$

$$\frac{dx}{2x} = \frac{4p^2 - 8p - 1}{p(1 + 4p^2)(1 - 4p^2)} dp$$

$$\int \frac{dx}{2x} = \int \frac{4p^2 - 8p - 1}{p(1 + 4p^2)(1 - 2p)(1 + 2p)} dp$$

$$\frac{A}{p} + \frac{Bp + C}{1 + 4p^2} + \frac{D}{1 - 2p} + \frac{E}{1 + 2p} = \frac{4p^2 - 8p - 1}{p(1 + 4p^2)(1 - 2p)(1 + 2p)}$$

$$A(1 + 4p^2)(1 - 4p^2) + (Bp + C)p(1 - 4p^2) + Dp(1 + 4p^2)(1 + 2p) +$$

$$+Ep(1 + 4p^2)(1 - 2p) = 4p^2 - 8p - 1$$

So, we have

$$-16A - 4B + 8D - 8E = 0$$

$$-4C+4D+4E=0$$

$$B+2D-2E=4$$

$$C+D+E=-8$$

$$A=-1$$

Then:

$$4-B+2D-2E=0$$

$$-C+D+E=0$$

$$B+2D-2E=4$$

$$C+D+E=-8$$

Solving the given system:

$$C=-4$$

$$B=4$$

$$\begin{cases} D=E \\ D+E=-4 \end{cases} \Rightarrow D=E=-2$$

So far:

$$\int \frac{dx}{2x}=\int\left(-\frac{1}{p}+4\cdot\frac{p-1}{1+4p^2}-\frac{2}{1-2p}-\frac{2}{1+2p}\right)dp$$

$$\int \frac{p-1}{1+4p^2}dp=\int\frac{d(p^2)}{2(1+4p^2)}dp-\int\frac{dp}{1+4p^2}=\frac{1}{8}\ln(1+4p^2)-\frac{1}{2}\tan^{-1}2p$$

Answer:

$$\ln 2x=-\ln p+\ln(1-2p)-\ln(1+2p)+\frac{1}{2}\ln(1+4p^2)-2\tan^{-1}2p+c$$

Question

2.

$$x^2p^2 + 3xyp + 2y^2 = 0$$

Solution

$$(xp + y)^2 + xyp + y^2 = 0$$

$$(xp + y)^2 + y(xp + y) = 0$$

$$(xp + y)(xp + 2y) = 0$$

Case 1:

$$xp + y = 0$$

$$p = \frac{dy}{dx} = -\frac{y}{x}$$

$$\ln y = -\ln x + \ln c$$

$$xy = c$$

Case 2:

$$xp + 2y = 0$$

$$p = \frac{dy}{dx} = -\frac{2y}{x}$$

$$\ln y = -2\ln x + \ln c$$

$$x^2y = c$$

Question

3.

$$x^2p^2 - 2p(xy - 2) + 2y^2 = 0$$

Solution

$$u = y, v = xy$$

$$\frac{dv}{dx} = xp + y$$

$$P = \frac{dv}{du} = \frac{xp + y}{p}$$

$$p = \frac{y}{P - x}$$

$$x^2p^2 - 2pxy + 4p + 2y^2 = 0$$

$$\frac{x^2y^2}{(P - x)^2} - \frac{2xy^2}{P - x} + \frac{4y}{P - x} + 2y^2 = 0$$

$$x^2y^2 + (4y - 2xy^2)(P - x) + 2y^2(P - x)^2 = 0$$

$$x^2y^2 - 4xy + 4Py + 2x^2y^2 - 2Pxy^2 + 2x^2y^2 - 4Pxy^2 + 2y^2P^2 = 0$$

$$-4xy + 4Py + 5x^2y^2 - 6Pxy^2 + 2y^2P^2 = 0$$

$$5v^2 - 4v + 4Pu - 6Puv - 2u^2P^2 = 0$$

The solution of this equation cannot be determined.