

Answer on Question # 79720 – Math – Differential Equations

Question

Solve the differential equation

$$(2x+y+3) \times (dy/dx) = (x+2y+1)$$

Solution

$$(2x+y+3) \times (dy/dx) = (x+2y+1)$$

$$\frac{dy}{dx} = \frac{x+2y+1}{2x+y+3}$$

$$\text{Or, } (2x+y+3) dy = (x+2y+1) dx$$

Let $M = (x+2y+1)$ and $N = (2x+y+3)$.

$$\text{Now, } \frac{\partial M}{\partial x} = 1 \text{ and } \frac{\partial N}{\partial y} = 1$$

So, it is an exact differential equation.

$$\text{Now, } \int M dx = \int (x + 2y + 1) dx = \frac{x^2}{2} + 2xy + x + p, \text{ where } p=p(y);$$

$$\text{Similarly, } \int N dy = \int (2x + y + 3) dy = 2xy + \frac{y^2}{2} + 3y + q, \text{ where } q=q(x)$$

Here p and q are integration constants.

Combining two expressions the solution is

$$\frac{x^2}{2} + 2xy + x + \frac{y^2}{2} + 3y = c,$$

where c is constant.

$$\text{Answer: } \frac{x^2}{2} + 2xy + x + \frac{y^2}{2} + 3y = c, \text{ where } c \text{ is constant.}$$