

## Answer on Question #79708 – Math – Statistics and Probability

### Question

A tennis coach believes that taller players are generally capable of letting faster serves. To investigate this hypothesis, he collects data on 20 adult male players he coaches. The height  $h$ , in a meter and the speed of each player faster serve,  $v$  in km per hour were recorded and summarized as follows

$$h=36.22$$

$$v=2275$$

$$h^2=65.7396$$

$$V^2=259853$$

$$HV=4128.03$$

1. Calculate the Pearson moment correlation coefficient for these data

### Solution

The Pearson moment correlation coefficient

$$r = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}}$$

$$n = 20;$$

$$\sum x_i y_i = HV = 4128.03;$$

$$\sum x_i = h = 36.22; \quad \sum y_i = v = 2275$$

$$\sum x_i^2 = h^2 = 65.7369 \quad \sum y_i^2 = V^2 = 259853$$

$$r = \frac{20 * 4128.03 - 36.22 * 2275}{\sqrt{20 * 65.7369 - (36.22)^2} * \sqrt{20 * 259853 - (2275)^2}} = \frac{82560.6 - 82400.5}{\sqrt{2.8496} * \sqrt{21435}} = 0.648$$

As we see, the correlation coefficient equals 64.8%. It shows a strong positive relationship, and taller players are generally capable of letting faster serves because its value is more than 60%.

**Answer:** A tennis coach believes that taller players are generally capable of letting faster serves because correlation coefficient equals 64.8%.

## Question

2. Comment on the which hypothesis

## Solution

### Method 1 The p-value approach

Let's specify the null and alternative hypotheses:

Null hypothesis **H<sub>0</sub>: r = 0**

Alternative hypothesis **HA: r ≠ 0**

Calculate the value of the test statistic using the following formula:

$$t^* = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{0.648 \times \sqrt{20-2}}{\sqrt{1-0.648^2}} = 3.61$$

Then we should use the resulting test statistic to calculate the P-value. To obtain the P-value, we need to compare the test statistic to a t-distribution with 18 degrees of freedom (since  $20 - 2 = 18$ ).

$t_{\text{distribution}} (\alpha=0.01) = 2.88$

So,  $t^* > t_{\text{distribution}} (\alpha=0.01)$ , we can reject the null hypothesis. There is a sufficient statistical evidence at the  $\alpha = 0.01$  level to conclude that taller players are generally capable of letting faster serves.

### Method 2 The Critical Values approach

Thus,  $r = 0.648$  using  $n = 20$ ,  $df = n - 2 = 20 - 2 = 18$ .

The critical values associated with  $df = 18$  are  $-0.5614$  and  $+0.5614$ .

If  $r <$  negative critical value or  $r >$  positive critical value, then  $r$  is significant.

Since  $r = 0.648$  and  $0.648 > 0.5614$ ,  $r$  is significant and we can surely conclude that taller players are generally capable of letting faster serves.

**Answer:** Using the results of 2 methods (a p-value and a table of Critical Values) we can conclude that taller players are generally capable of letting faster serves.