

Answer on Question #79704 – Math – Combinatorics | Number Theory

Question

Sayem is counting the minimum number of lines m , that can be drawn on the plane so that they interact in exactly 200 distinct points. What is m ?

Solution

We obtain the minimum number of lines when there will be the most intersections, i.e. when each line intersects with each other line one time.

The first line can't intersect anything.

The second line intersects the first one: +1 intersections.

The third line intersects both of the first two: +2 intersections.

The fourth line intersects the three already there: +3 intersections.

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The m -th line intersects the $m-1$ already there: $+(m-1)$ intersections.

So, the m lines have $N = 1 + 2 + 3 + \dots + m - 1 = \frac{m(m-1)}{2}$ intersections (here the formula for the sum of $(m-1)$ terms of the arithmetic sequence was used).

If $N = 200$ then $\frac{m(m-1)}{2} = 200$, hence $m = 21$.

Answer: 21.