Answer on Question #79650 – Math – Differential Equations

Question

$(x^2 + y^2)dx - 2xydy = 0.$

   a. $x + \dfrac{y}{x} = c$

   b. $x - \dfrac{y}{x} = c$

   c. $x - \dfrac{y^2}{x} = c$

   d. $x + \dfrac{y^2}{x} = c$

Solution

This differential equation is homogeneous.

Replace $y = ux, dy = udx + xdu$. Here $u = u(x)$.

$$(x^2 + (ux)^2)dx - 2xux(udx + xdu) = 0$$

$$x^2(1 + u^2)dx - 2x^2u^2dx - 2x^3udu = 0$$

$$x^2(1 - u^2)dx - 2x^3udu = 0$$

$$\frac{x^2}{x^3}dx = \frac{2u}{1 - u^2}du$$

$$\frac{1}{x}dx = -\frac{1}{1 - u^2}d(1 - u^2)$$

$$\ln(x) = -ln(1 - u^2) + \ln(c)$$

$$x(1 - u^2) = c$$

Replace: $u = \dfrac{y}{x}$

$$x\left(1 - \frac{y^2}{x^2}\right) = c$$

$$x - \frac{y^2}{x} = c$$

Answer: c. $x - \dfrac{y^2}{x} = c$.