Question 1. *A subspace Y of a normed space X is said to be invariant under a linear operator $T : X \to X$ if $T(y) \in Y$. Let $\lambda \in \sigma_p(T)$ (λ belongs to the point spectrum), $T \in B(X, X)$, X be a complex Banach space. Show that the eigenspace of λ is T-invariant.*

Solution. Recall that the point spectrum of T is the set of all eigenvalues of T, i. e. all $\lambda \in \mathbb{C}$ such that $Tx = \lambda x$ for some $x \in X$. For the fixed eigenvalue λ the set of such vectors x forms a linear subspace of X, which is called the eigenspace of λ.

Let Y be the eigenspace of λ. Show that Y is T-invariant. Take $y \in Y$ and show that $Ty \in Y$. Indeed, $Ty = \lambda y \in Y$, because Y is a subspace of X. $\square$