The lifetime of a car batteries is normally distributed with mean 1 year and standard deviation 1.5 months.

Question

a) calculate the probability that a randomly selected car battery will last 10 to 11 months.

Solution

$$z = \frac{X - \mu}{\sigma};$$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{10 - 12}{1.5} = -1.33$$

$$z_2 = \frac{11 - 12}{1.5} = -0.67$$

$$p(x_1 < x < x_2) = p(z_1 < z < z_2) = p(z < z_2) - p(z < z_1);$$

$$p(z < z_1) = 0.0912$$

$$p(z < z_2) = 0.2525$$

$$p(x_1 < x < x_2) = 0.2525 - 0.0912 = 0.1613.$$

Question

b) calculate the minimum lifetime of the 30% longest lasting car batteries

Solution

$Z_{0.3} = 0.52;$

$X = \mu + z\sigma = 12 + 0.52 \times 1.5 = 12.78.$

Question

c) by improving the quality of the car batteries, the lifetime is to be increased such that only 2.5% of the car batteries will fail before 10 months. Find the new mean lifetime of the car batteries, Assume the standard deviation remains unchanged,

Solution

$Z_{0.025}$ = -1.96

$x = \mu + z\sigma$; hence $\mu = x - z\sigma = 10 - (-1.96) \times 1.5 = 12.94$.