Answer on Question #79558 – Math – Trigonometry

Question

If sin4B + sin2B =1 then prove that tan4B - tan2B =1.

Solution

The question is incorrect. Let's prove it. Assume that there exists B such that sin4B + sin 2B = 1 and tan4B – tan2B = 1.

Let sin2B = x and cos2B = y. Then we have: sin4B = 2sin2Bcos2B = 2xy;

Tan2B = x/y; tan4B = $\frac{2\tan 2B}{1-\tan^2 2B}$ = $\frac{2xy}{y^2-x^2}$.

Then, we have: 2xy + x = 1 (**) and $\frac{2xy}{y^2-x^2} - \frac{x}{y}$ = 1, with $x^2 + y^2 = 1$.

The second condition is equivalent to $2xy^2 = (x+y)(y^2-x^2) = (x+y)^2(y-x)$. (*)

Note that: $(x+y)^2 = x^2+2xy+y^2 = 1+2xy=1+(1-x) = 2-x$.

Also, $2xy^2 = (1-x)y = y-xy$.

Then, from the (*), we get: $y- xy = (2-x)(y-x)=2y-xy-2x+x^2$,

Which is equivalent to: $y=2x - x^2$.

On the other hand, (from **) we get that y = $\frac{1-x}{2x}$.

Thus, $2x-x^2 = \frac{1-x}{2x}$, so $1-x = 4x^2 - 2x^3$, and then $2x^3-4x^2-x+1=0$. (°)

Since $x^2 + y^2 = 1$, we get: $x^2 +(\frac{1-x}{2x})^2 = 1$, which means that $4x^4+1-2x+x^2=4x^2$, so $4x^4-3x^2-2x+1=0$,

which is equivalent to $(x-1)(4x^3+4x^2+x-1)=0$. Since x = 1 cannot be the root (because otherwise y=0 and tan2B does not exist), we have $4x^3+4x^2+x-1=0$.

Sum it up with (°): $6x^3 = 0$, which means that x = 0, y = ± 1. But I this case, the equation (**) is wrong.

Thus, the task is incorrect.