Answer on Question #79476 – Math – Differential Geometry | Topology

Question

if A = 5t2 + tj-t3k and B = sin t1-costj evaluate d/dt (AxB)

Solution

the derivative of the cross product of two vector functions is

$$\frac{d}{dt}(\vec{A}\times\vec{B}) = \frac{d\vec{A}}{dt}\times\vec{B} + \vec{A}\times\frac{d\vec{B}}{dt}$$

$$\vec{A} = a_1(t)\vec{\imath} + a_2(t)\vec{\jmath} + a_3(t)\vec{k} = 5t^2\vec{\imath} + t\vec{\jmath} - t^3\vec{k}$$

$$\frac{d\vec{A}}{dt} = \frac{da_1(t)}{dt}\vec{\imath} + \frac{da_2(t)}{dt}\vec{\jmath} + \frac{da_3(t)}{dt}\vec{k} = 10t\vec{\imath} + \vec{\jmath} - 3t^2\vec{k}$$

$$\vec{B} = b_1(t)\vec{\imath} + b_2(t)\vec{\jmath} + b_3(t)\vec{k} = sin(t)\,\vec{\imath} - cos(t)\vec{\jmath}$$

$$\frac{d\vec{B}}{dt} = \frac{db_1(t)}{dt}\vec{\imath} + \frac{db_2(t)}{dt}\vec{\jmath} + \frac{db_3(t)}{dt}\vec{k} = cos(t)\vec{\imath} + sin(t)\,\vec{\jmath}$$

cross product of two vectors u × v:

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = (u_2v_3 - u_3v_2)\vec{\imath} + (u_3v_1 - u_1v_3)\vec{\jmath} + (u_1v_2 - u_2v_1)\vec{k}$$

$$\frac{d\vec{A}}{dt}\times\vec{B} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 10t & 1 & -3t^2 \\ \sin(t) & -\cos(t) & 0 \end{vmatrix}$$

$$= [1\cdot 0 - (-3t^2)(-cos(t))]\vec{\imath} + [(-3t^2)\,sin(t) - 10t\cdot 0]\vec{\jmath}$$
$$+ [10t(-cos(t)) - 1\cdot sin(t)]\vec{k}$$
$$= -3t^2cos(t)\vec{\imath} - 3t^2\,sin(t)\,\vec{\jmath} - (10t\cdot cos(t) + sin(t))\vec{k}$$

$$\vec{A}\times\frac{d\vec{B}}{dt} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 5t^2 & t & -t^3 \\ \cos(t) & \sin(t) & 0 \end{vmatrix}$$

$$= [t\cdot 0 - (-t^3)sin(t)]\vec{\imath} + [(-t^3)\,cos(t) - 5t^2\cdot 0]\vec{\jmath} + [5t^2sin(t) - t\cdot cos(t)]\vec{k}$$
$$= t^3sin(t)\vec{\imath} - t^3\,cos(t)\,\vec{\jmath} + (5t^2sin(t) - t\cdot cos(t))\vec{k}$$

So,

$$\frac{d}{dt}\left(\vec{A}\times\vec{B}\right)=\frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}$$
$$=[t^3sin(t)-3t^2cos(t)]\vec{\imath}-[t^3\,cos(t)+3t^2\,sin(t)]\vec{\jmath}$$
$$+[5t^2sin(t)-t\,cos(t)-10t\cdot cos(t)-sin(t)]\vec{k}$$
$$=t^2\left(t\cdot sin(t)-3cos(t)\right)\vec{\imath}-t^2(t\cdot cos(t)+3\,sin(t))\,\vec{\jmath}$$
$$+\left((5t^2-1)sin(t)-11t\cdot cos(t)\right)\vec{k}$$

Answer: $\frac{d}{dt}\left(\vec{A}\times\vec{B}\right)=t^2\left(t\cdot sin(t)-3cos(t)\right)\vec{\imath}-t^2(t\cdot cos(t)+3\,sin(t))\,\vec{\jmath}+\left((5t^2-1)sin(t)-11t\cdot cos(t)\right)\vec{k}$