Answer on Question #79450 – Math – Statistics and Probability

Question

Empty houses According to the 2010 Census, 11.4% of all housing units in the United States were vacant. A county supervisor wonders if her county is different from this. She randomly selects 850 housing units in her county and finds that 129 of the housing units are vacant.

Solution

We have to test the null hypothesis H_0 : the county has 11.4% vacant houses. .

Let

$$X_i = \begin{cases} 1, if\ i-th\ house\ is\ vacant, \\ 0, otherwise \end{cases}$$

Then under null hypothesis

$$\bar{x} = EX_i = 0.114 , \ \sigma^2 = \mathrm{var}(X_i) = 0.114 \cdot (1 - 0.114) = 0.101$$

$$\frac{\sum\limits_{i=1}^{850} X_i - 850\bar{x}}{\sqrt{850 \cdot 0.101}} = \frac{\sum\limits_{i=1}^{850} X_i - 96.9}{9.27} \sim N(0,1)$$

Find p-value under H_0 :

$$P_{H_0}\left(\left| \sum_{i=1}^{850} X_i - 96.9 \right| \geq |129 - 96.9| \right) = P_{H_0}\left(\frac{\left| \sum\limits_{i=1}^{850} X_i - 96.9 \right|}{9.27} \geq 3.46 \right) = 2 \cdot (1 - F(3.46)) =$$

$$= 2 \cdot (1 - 0.99973) = 0.00054$$

Since this value is very small (at least less than usual $\alpha = 0.05$) we reject the null hypothesis. So the conclusion is that her county is different from this.