

ANSWER on Question #79414 – Math – Differential Equations

QUESTION

Solve the differential equation

$$\frac{dy}{dx} = \frac{2y^2 + 3xy}{x^2}$$

Possible answers:

a)

$$y = \frac{cx^3}{1 - cx^2}$$

b)

$$y = \frac{cx^3}{1 + cx^2}$$

c)

$$y = -\frac{cx^3}{1 - cx^2}$$

d)

$$y = -\frac{cx^3}{1 + cx^2}$$

SOLUTION

$$\frac{dy}{dx} = \frac{2y^2 + 3xy}{x^2} \rightarrow \frac{dy}{dx} = \frac{2y^2}{x^2} + \frac{3xy}{x^2} \rightarrow \boxed{\frac{dy}{dx} = 2 \cdot \left(\frac{y}{x}\right)^2 + 3 \cdot \frac{y}{x}}$$

We introduce the substitution

$$u = \frac{y}{x} \rightarrow y = ux \rightarrow \frac{dy}{dx} = \frac{du}{dx} \cdot x + u \cdot 1 \rightarrow \boxed{\frac{dy}{dx} = \frac{du}{dx} \cdot x + u}$$

Then,

$$\begin{cases} \frac{dy}{dx} = 2 \cdot \left(\frac{y}{x}\right)^2 + 3 \cdot \frac{y}{x} \\ u = \frac{y}{x} \\ \frac{dy}{dx} = \frac{du}{dx} \cdot x + u \end{cases} \rightarrow \frac{du}{dx} \cdot x + u = 2u^2 + 3u \rightarrow \frac{du}{dx} \cdot x = 2u^2 + 3u - u \rightarrow \frac{du}{dx} \cdot x = 2u^2 + 2u \rightarrow$$

$$\frac{du}{dx} \cdot x = 2u^2 + 2u \left| \times \left(\frac{2 \cdot dx}{x \cdot (2u^2 + 2u)} \right) \rightarrow \frac{2 \cdot du}{2u^2 + 2u} = \frac{2 \cdot dx}{x} \rightarrow \frac{2 \cdot ((u+1) - u)du}{2u(u+1)} = \frac{2 \cdot dx}{x} \rightarrow$$

$$\left(\frac{1}{u} - \frac{1}{u+1} \right) du = \left(\frac{2}{x} \right) dx \rightarrow \int \left(\frac{1}{u} - \frac{1}{u+1} \right) du = \int \left(\frac{2}{x} \right) dx \rightarrow \ln|u| - \ln|u+1| = 2 \cdot \ln|x| + \ln|c| \rightarrow$$

$$\ln \left| \frac{u}{u+1} \right| = \ln|cx^2| \rightarrow \frac{u}{u+1} = cx^2 \left| \times (u+1) \rightarrow u = cx^2 \cdot (u+1) \rightarrow u = cx^2 \cdot u + cx^2 \rightarrow$$

$$u - cx^2 \cdot u = cx^2 \rightarrow u(1 - cx^2) = cx^2 \rightarrow \boxed{u = \frac{cx^2}{1 - cx^2}}$$

We recall that we introduced a substitution

$$u = \frac{y}{x}$$

Then,

$$\begin{cases} u = \frac{cx^2}{1 - cx^2} \\ u = \frac{y}{x} \end{cases} \rightarrow \frac{y}{x} = \frac{cx^2}{1 - cx^2} \left| \times (x) \rightarrow y = \frac{cx^3}{1 - cx^2}$$

Conclusion,

$$\frac{dy}{dx} = \frac{2y^2 + 3xy}{x^2} \rightarrow y = \frac{cx^3}{1 - cx^2} - \text{ANSWER a)}$$

ANSWER:

$$a) y = \frac{cx^3}{1 - cx^2}$$