Answer on Question #79369 – Math – Differential Equations

Question

Which of the following satisfied the Laplace's equation in the plane

1. $x^2 + y^2$

2. $x^2 - y^2$

3. $x + y$

4. $x - y$

Solution

The two-dimensional (planar) Laplace's equation has the following form:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0. \tag{1}$$

Substituting the given solutions into (1), we obtain:

$$u(x, y) = x^2 + y^2 \;\to\; 2+2=4 \neq 0,$$

$$u(x, y) = x^2 - y^2 \;\to\; 2-2=0,$$

$$u(x, y) = x + y \;\to\; 0+0=0, \tag{2}$$

$$u(x, y) = x - y \;\to\; 0-0=0.$$

Hence the functions $x^2 - y^2$, $x + y$ and $x - y$ satisfy the Laplace's equation in the plane.

Answer: *2. $x^2 - y^2$; 3. $x + y$; 4. $x - y$.*