Question

Check whether the set of integers is countable or not.

Solution

The set of natural integers is countable. Let N denote the set of integers,

$Z=\{0, 1, 2, 3,…\}$ denote the set of natural numbers together with 0.

To prove that the set N of integers is countable it is sufficient to construct a bijection

$$f: N \to Z.$$

We construct it in such a way:

$$f(0)=0, f(1)=1, f(2)=-1, f(3)=2, f(4)=-2, f(5)=3, …$$

The formula for this function is

$$f(n) = (-1)^{n+1} \left\lceil \frac{n}{2} \right\rceil,$$

where $\lceil \cdot \rceil$ denotes the ceiling function.

The inverse map is

$$f^{-1}(k) = 2|k| - 1_{(0,+\infty)}(k).$$

That's why this function is really a bijection.

Answer: the set of integers is countable.