Answer on Question #79346 - Math - Calculus

(a)

$$G'(t) = k - rG(t).$$

(b) Consider the homogeneous equation

$$G'(t) = -rG(t).$$

The general solution of this equation is $G(t) = Ce^{-rt}$ where C is an arbitrary constant. It easy to see that a particular solution of the equation is $G(t) = Cc^{-1}$ where C is an arbitrary constant. It easy to see that a particular solution of the equation G'(t) = k - rG(t) is $G(t) = \frac{k}{r}$. Thus the general solution of the equation G'(t) = k - rG(t) is $G(t) = \frac{k}{r} + Ce^{-rt}$. From the initial condition G(0) = 0 we have $C = -\frac{k}{r}$. Finally $G(t) = \frac{k}{r} - \frac{k}{r}e^{-rt}$. (c) From the general solution formula of the equation G'(t) = k - rG(t) it is easy to see

that $\lim_{t\to\infty} G(t) = \frac{k}{r}$ for any initial condition.