Answer on Question # 79329 – Math – Differential Equations

Question

Solve the differential equation:

$(x^2 + y^2)\,dx - 2xy\,dy = 0$

Solution

$(x^2 + y^2)\,dx - 2xy\,dy = 0$

Or, $(x^2 + y^2) - 2xy\frac{dy}{dx} = 0$

Or, $\frac{dy}{dx} = \frac{(x^2+y^2)}{2xy}$ ………………. (1)

Let $y = vx$ ……………………(2) [v is the function of x]

Now differentiate equation (2) with respect to x and we get,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now, put the values of $\frac{dy}{dx}$ and $y = vx$ in equation (1), we get,

$v + x\frac{dv}{dx} = \frac{(1+v^2)}{2v}$

or, $x\frac{dv}{dx} = \frac{(1+v^2)}{2v} - v = \frac{(1-v^2)}{2v}$

or, $-\frac{2v}{v^2-1}dv = \frac{1}{x}dx$ …………………………….(3)

Now, integrating both sides of equation (3), we get,

$\ln\left(\frac{1}{v^2-1}\right) = \ln x + \ln p$ [where ln p is integration constant]

or, $\ln\frac{x^2}{(y^2-x^2)} = \ln(x\,p)$

or, $y^2 - x^2 = x\left(\frac{1}{p}\right) = c\,x$ [where, $c = \frac{1}{p}$ = constant]

Answer: Solution is $y^2 - x^2 = c\,x$