

**Answer on Question #79308 – Math – Differential Equations
Question**

Integration factor

1. $(x + 2)dx + xdy = 0$
2. $(x^2 + y^2 + x)dx + ydy = 0$
3. $(5y - 6x)dx + xdy = 0$
4. $(3x - y^2)dx - 4xydy = 0$

Solution

$$M(x, y)dx + N(x, y)dy = 0$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Integration factor $u(x, y)$

$$\frac{\partial(uN(x, y))}{\partial x} = \frac{\partial(uM(x, y))}{\partial y}$$

1. $(x + 2)dx + xdy = 0$

$$M(x, y) = x + 2, \quad \frac{\partial M}{\partial y} = 0$$

$$N(x, y) = x, \quad \frac{\partial N}{\partial x} = 1$$

$$0 \neq 1 \Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{0 - 1}{x} = -\frac{1}{x}$$

Then the integration factor $u(x)$ is function of x only

$$u(x) = \exp\left(\int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx\right) = \exp\left(\int \left(-\frac{1}{x}\right) dx\right) = \exp(-\ln|x|) = \frac{1}{x}$$

The integration factor

$$u = \frac{1}{x}$$

$$2. (x^2 + y^2 + x)dx + ydy = 0$$

$$M(x, y) = x^2 + y^2 + x, \quad \frac{\partial M}{\partial y} = 2y$$

$$N(x, y) = y, \quad \frac{\partial N}{\partial x} = 0$$

$$2y \neq 0 \Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y - 0 = 2y$$

$$z = x^2 + y^2, \quad \frac{\partial z}{\partial x} = 2x, \quad \frac{\partial z}{\partial y} = 2y$$

$$\frac{1}{u} \left(\frac{du}{dz} \right) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N \frac{\partial z}{\partial x} - M \frac{\partial z}{\partial y}} = \frac{2y}{y(2x) - (x^2 + y^2 + x)(2y)} =$$

$$= \frac{1}{x - x^2 - y^2 - x} = -\frac{1}{x^2 + y^2} = -\frac{1}{z}$$

$$\frac{du}{u} = -\frac{dz}{z}$$

$$\int \frac{du}{u} = -\int \frac{dz}{z}$$

$$u = \frac{1}{z}$$

The integration factor

$$u = \frac{1}{x^2 + y^2}$$

$$3. (5y - 6x)dx + xdy = 0$$

$$M(x, y) = 5y - 6x, \quad \frac{\partial M}{\partial y} = 5$$

$$N(x, y) = x, \quad \frac{\partial N}{\partial x} = 1$$

$$5 \neq 1 \Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{5 - 1}{x} = \frac{4}{x}$$

Then the integration factor $u(x)$ is function of x only

$$u(x) = \exp\left(\int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx\right) = \exp\left(\int \left(\frac{4}{x}\right) dx\right) = \exp(4 \ln|x|) = x^4$$

$$u(x) = x^4$$

$$4. (3x - y^2)dx - 4xydy = 0$$

$$M(x, y) = 3x - y^2, \quad \frac{\partial M}{\partial y} = -2y$$

$$N(x, y) = -4xy, \quad \frac{\partial N}{\partial x} = -4y$$

$$-2y \neq -4y \Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -2y - (-4y) = 2y$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y}{-4xy} = -\frac{1}{2x}$$

Then the integration factor $u(x)$ is function of x only

$$u(x) = \exp\left(\int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx\right) = \exp\left(\int \left(-\frac{1}{2x}\right) dx\right) = \exp\left(-\frac{1}{2} \ln|x|\right) = \frac{1}{\sqrt{x}}$$

The integration factor

$$u = \frac{1}{\sqrt{x}}$$