## Answer on Question #79287 – Math – Calculus

## Question

Find the area enclosed by the curve $r = a(1-\cos\theta)$.

## Solution

The area enclosed by the curve is calculated by the formula

$$S = \frac{1}{2}\cdot\int_{\alpha}^{\beta} r^2(\theta)d\theta$$

$r \geq 0,\ \ a(1-\cos\theta) \geq 0$

If $a \geq 0$, that $1-\cos\theta \geq 0 \Rightarrow \cos\theta \leq 1 \Rightarrow 2\pi n \leq \theta \leq 2\pi + 2\pi n, n \in Z \Rightarrow 0 \leq \theta \leq 2\pi$

If $a < 0$, that $1-\cos\theta < 0 \Rightarrow \cos\theta > 1$ no solution

$$S = \frac{1}{2}\cdot\int_0^{2\pi} a^2(1-\cos\theta)^2 d\theta = \frac{a^2}{2}\cdot\int_0^{2\pi}(1-2\cos\theta+\cos^2\theta)d\theta = \frac{a^2}{2}\cdot\int_0^{2\pi}(1-2\cos\theta+\frac{1+\cos 2\theta}{2})d\theta =$$

$$= \frac{a^2}{2}\cdot(\theta - 2\sin\theta + \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta)\Big|_0^{2\pi} = \frac{a^2}{2}\cdot(2\pi - 2\sin 2\pi + \frac{1}{2}\cdot 2\pi + \frac{1}{4}\sin 4\pi) - 0 =$$

$$= \frac{a^2}{2}\cdot(2\pi - 0 + \pi + 0) = \frac{3\pi\cdot a^2}{2}$$

**Answer:** $\frac{3\pi\cdot a^2}{2}$