Answer on Question #79286 – Math – Calculus

Question

Intravenous infusion of glucose into the blood-stream of a patient is an important medical technique. To study this process, let G(t) be the amount of glucose in the patient's blood stream t minutes after the process begins. Assume that glucose is infused into the bloodstream at a constant rate of k(in g/min). Also assume that at the same time, the glucose is converted and removed from the bloodstream at a rate proportional to the amount of glucose still present, with the proportionality constant r.

Solution

The rate of change of glucose due to infusion is given by k. The rate of change of glucose due to the conversion is given by $-rG(t)$. Thus the total rate of change of $G(t)$ is given by

$$G'(t) = k - rG(t)$$

Since k and r are constants, the solution of this equation is easy to find:

$$G(t) = Ce^{-rt} + \frac{k}{r},$$

where C is some constant.

In order to find C we must consider the case $t = 0$:

$$G(0) = C + \frac{k}{r}$$

$$C = G(0) - \frac{k}{r}$$

Thus we get

$$G(t) = G(0)e^{-rt} + \frac{k}{r}(1 - e^{-rt})$$

Answer: $G(t) = G(0)e^{-rt} + \frac{k}{r}(1 - e^{-rt})$.