Answer on Question #79276 – Math – Differential Equations

Question

Solve the initial value problem $(1+y^2)dx+(1+x^2)dy=0$, $y(0)=-1$

Solution

This equation with separating variables:

$$(1+y^2)dx+(1+x^2)dy=0$$

$$(1+y^2)dx=-(1+x^2)dy$$

$$\frac{dx}{1+x^2}=-\frac{dy}{1+y^2}$$

We integrate both sides of equation:

$$\int\frac{dx}{1+x^2}=\int-\frac{dy}{1+y^2}$$

$$\tan^{-1}x=-\tan^{-1}y+C$$

$$\tan^{-1}0=-\tan^{-1}(-1)+C$$

$$0=-(-\frac{\pi}{4})+C$$

$$C=-\frac{\pi}{4}$$

We substitute the obtained constant in equation:

$$\tan^{-1}x=-\tan^{-1}y-\frac{\pi}{4}$$

$$\tan^{-1}x+\tan^{-1}y=-\frac{\pi}{4}$$

Answer: $\tan^{-1}x+\tan^{-1}y=-\frac{\pi}{4}$.