

Answer on Question #79270 – Math – Analytic Geometry

Question

1. The sides of a triangle are given by the equations $2x + y = 5$, $x + 2y = 7$ and $x - y = 1$, find the vertices of the triangle and illustrate with a sketch.
2. Show that the three lines given by the equations $3x + 5y + 7 = 0$, $x + 2y + 2 = 0$ and $2x - y + 9 = 0$ are concurrent (i. e. pass through the one point)

Solution

1. The vertices of a triangle can be found as the points of intersection between each pair of the lines, i.e. the following set of equations should be solved:

$$\begin{cases} 2x + y = 5 \\ x + 2y = 7 \end{cases} \quad (1)$$

$$\begin{cases} 2x + y = 5 \\ x - y = 1 \end{cases} \quad (2)$$

$$\begin{cases} x + 2y = 7 \\ x - y = 1 \end{cases} \quad (3)$$

Solving the system of equations (1), (2) and (3) we obtain the coordinates (1,3), (2,1) and (3,2) correspondingly (note: coordinates of the vertices are written in the form (x_i, y_i)). Now it is possible to make the sketch (see fig.1).

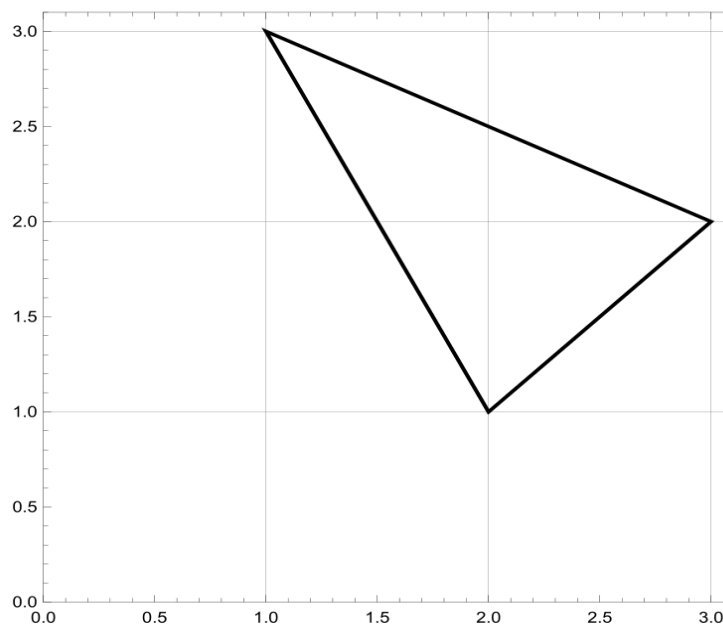


Figure 1.

2. To prove that the given lines are concurrent we need to find the point of their intersection. This can be achieved by finding the solution (if exists) of the following system of equations:

$$\begin{cases} 3x + 5y + 7 = 0 \\ x + 2y + 2 = 0 \\ 2x - y + 9 = 0 \end{cases} \quad (4)$$

It can be easily shown that the point $(-4,1)$ is the solution of the system (4). Hence, the given lines are concurrent.