Answer on Question #79269 – Math – Analytic Geometry

Question

1. Find the point of intersection of the following pairs of lines whose equations are given.

a) $x + 3y = 9$ and $5x - 2y = 11$

b) $4x + 3y = 8$ and $6x - 2y = -14$

c) $3x + 2y - 7 = 0$ and $5x - 6y = 7$

Solution

We will find the point of intersection of the following pairs of lines whose equations are given if we make a system of two equations and solve it.

a) $x + 3y = 9$ and $5x - 2y = 11$.

The system of equations:

$$\begin{cases} x + 3y = 9, \\ 5x - 2y = 11. \end{cases}$$

Solve the system of equations by substitution method. For this solve the first equation for x:

$$x = 9 - 3y.$$

Substitute the expression $9 - 3y$ for x into the second equation:

$$\begin{cases} x = 9 - 3y, \\ 5(9 - 3y) - 2y = 11. \end{cases}$$

Solve the second equation for y:

$$\begin{cases} x = 9 - 3y, \\ 45 - 15y - 2y = 11; \end{cases}$$

$$\begin{cases} x = 9 - 3y, \\ -17y = -34; \end{cases}$$

$$\begin{cases} x = 9 - 3y, \\ y = 2. \end{cases}$$

Plug in 2 for y into the equation $x = 9 - 3y$ to find x's value.

$$x = 9 - 3 \cdot 2,$$

$$x = 3.$$

Check the proposed ordered pair solution in both original equations.

We find that if we plug the ordered pair (3; 2) into both equations of the original system, that this is a solution to both of them.

(3; 2) is a solution to our system.

Answer: (3; 2) is the point of intersection of the pairs of lines whose equations are x + 3y = 9 and 5x - 2y = 11.

b) 4x + 3y = 8 and 6x - 2y = - 14.

The system of equations:

$$\begin{cases} 4x + 3y = 8, \\ 6x - 2y = -14. \end{cases}$$

Solve the system of equations by elimination method. Multiply the first equation by 2 and the second equation by 3:

$$\begin{cases} 4x + 3y = 8, \\ 6x - 2y = -14. \end{cases} \begin{matrix} | 2 \\ | 3 \end{matrix}$$

We get:

$$\begin{cases} 8x + 6y = 16, \\ 18x - 6y = -42. \end{cases}$$

Add equations:

$$26x = -26.$$

Solve for x:

$$x = -1.$$

Simplify the second equation:

$$2y = 6x + 14;$$

$$y = \frac{1}{2}(6x + 14).$$

Plug in -1 for x into the second simplified equation to find y's value:

$$y = \frac{1}{2}(6 \cdot (-1) + 14),$$

$$y = 4.$$

Check the proposed ordered pair solution in both original equations.

We find that if we plug the ordered pair (-1; 4) into both equations of the original system, that this is a solution to both of them.

(-1; 4) is a solution to our system.

Answer: (-1; 4) is the point of intersection of the pairs of lines whose equations are 4x + 3y = 8 and 6x - 2y = - 14.

c) 3x + 2y - 7 = 0 and 5x - 6y = 7.

The system of equations:

$$\begin{cases} 3x + 2y - 7 = 0, \\ 5x - 6y = 7. \end{cases}$$

Solve the system of equations by elimination method. Multiply the first equation by 3 and simplify it:

$$\begin{cases} 3x + 2y - 7 = 0, \\ 5x - 6y = 7. \end{cases} \Big| \; 3$$

We get:

$$\begin{cases} 9x + 6y = 21, \\ 5x - 6y = 7. \end{cases}$$

Add equations:

$$14x = 28.$$

Solve for x:

$$x = 2.$$

Simplify the first equation:

$$2y = -3x + 7;$$

$$y = \frac{1}{2}(-3x + 7).$$

Plug in 2 for x into the first simplified equation to find y's value:

$$y = \frac{1}{2}(-3 \cdot 2 + 7),$$

$$y = \frac{1}{2}.$$

Check the proposed ordered pair solution in both original equations.

We find that if we plug the ordered pair $(2; \frac{1}{2})$ into both equations of the original system, that this is a solution to both of them.

$(2; \frac{1}{2})$ is a solution to our system.

Answer: $(2; \frac{1}{2})$ is the point of intersection of the pairs of lines whose equations are 3x + 2y - 7 = 0 and 5x - 6y = 7.

Question

2. Find the equation of the straight line which passes through the origin and through the point of intersection of the lines 4x - y- 3 = 0 and x + 2y - 12 = 0.

Solution

Find the point of intersection of the lines 4x - y- 3 = 0 and x + 2y - 12 = 0.

Make a system of two equations and solve it:

$$\begin{cases} 4x - y - 3 = 0, \\ x + 2y - 12 = 0. \end{cases}$$

Simplify both equations:

$$\begin{cases} 4x - y = 3, \\ x + 2y = 12. \end{cases}$$

Solve the system of equations by elimination method. Multiply the first equation by 2:

$$\begin{cases} 4x - y = 3, \\ x + 2y = 12. \end{cases} \Big| \; 2$$

We get:

$$\begin{cases} 8x - 2y = 6, \\ x + 2y = 12. \end{cases}$$

Add equations:

$$9x = 18.$$

Solve for x:

$$x = 2.$$

Simplify the first equation:

$$y = 4x - 3.$$

Plug in 2 for x into the first simplified equation to find y's value:

$$y = 4 \cdot 2 - 3,$$

$$y = 5.$$

Check the proposed ordered pair solution in both original equations.

We find that if we plug the ordered pair $(2;\ 5)$ into both equations of the original system, that this is a solution to both of them.

$(2; 5)$ is a solution to our system.

$(2;\ 5)$ is the point of intersection of the pairs of lines 4x - y- 3 = 0 and

x + 2y - 12 = 0.

Write a formula for the equation of a line from 2 points:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}.$$

Find the equation of a line from 2 points: (2; 5) and (0; 0).

$$\frac{x - 2}{-2} = \frac{y - 5}{-5}.$$

Solve this equation:

$$-5x + 10 = -2y + 10,$$

$$-5x + 2y = 0.$$

Answer: $-5x + 2y = 0$ is the equation of the straight line which passes through the origin and through the point of intersection of the lines 4x - y- 3 = 0 and x + 2y - 12 = 0.