Answer on Question #79261 – Math – Trigonometry

Question

Prove that $\sec\left(\frac{3\pi}{2}-\theta\right)\cdot \sec\left(\theta-\frac{5\pi}{2}\right)+\tan\left(\frac{5\pi}{2}+\theta\right)\cdot\tan\left(\theta-\frac{3\pi}{2}\right)=(-1)$

Solution

$$\text{L.H.S} = \sec\left(\frac{3\pi}{2}-\theta\right)\cdot \sec\left(\theta-\frac{5\pi}{2}\right)+\tan\left(\frac{5\pi}{2}+\theta\right)\cdot\tan\left(\theta-\frac{3\pi}{2}\right)$$

$$= \sec\left(\frac{3\pi}{2}-\theta\right)\cdot \sec\left[-\left(\frac{5\pi}{2}-\theta\right)\right]+\tan\left(\frac{5\pi}{2}+\theta\right)\cdot\tan\left[-\left(\frac{3\pi}{2}-\theta\right)\right]$$

$$= \sec\left(\frac{3\pi}{2}-\theta\right)\cdot \sec\left(\frac{5\pi}{2}-\theta\right)+\tan\left(\frac{5\pi}{2}+\theta\right)\cdot\left[-\tan\left(\frac{3\pi}{2}-\theta\right)\right]$$

$$=[-\operatorname{cosec}\theta]\cdot[\operatorname{cosec}\theta]+[-\cot\theta]\cdot[-\cot\theta]$$

$$= -\operatorname{cosec}^2\theta+\cot^2\theta$$

$$= -1 \qquad \left[\text{As, } \operatorname{cosec}^2\theta-\cot^2\theta=\frac{1}{\sin^2\theta}-\frac{\cos^2\theta}{\sin^2\theta}=\frac{1-\cos^2\theta}{\sin^2\theta}=\frac{\sin^2\theta}{\sin^2\theta}=1\right]$$

$$= \text{R.H.S}$$

Answer: $\sec\left(\frac{3\pi}{2}-\theta\right)\cdot\sec\left(\theta-\frac{5\pi}{2}\right)+\tan\left(\frac{5\pi}{2}+\theta\right)\cdot\tan\left(\theta-\frac{3\pi}{2}\right)=-1$ has been proved.