Answer on Question #79260 – Math – Algebra

Question

Write an odd natural number as a sum of two integers m1 and m2 in a way that m1m2 is maximum. Solve using inequalities

Solution

Let the odd number be $2n + 1$
and let us divide it in two numbers x and $2n + 1 - x$
then their product is $2nx + x - x^2$
The product will be maximum if dydx=0, where

$$y = f(x) = 2nx + x - x^2$$

and hence for maxima $\frac{dy}{dx} = 2n + 1 - 2x = 0$
or $x = \frac{2n+1}{2} = n + \frac{1}{2}$
but as $2n + 1$ is odd, x is a fraction.
But as x has to be an integer, we can have the integers as n and $n + 1$ i.e. one integer just less than half the number and other integer just more than half the number. If the number is $2n + 1$, the numbers are n and $n + 1$.
For example, if number is 37, the two numbers m_1 and m_2 would be 18 and 19 and their product 342 would be the maximum one can have if 37 is split in two integers.

Answer:
One integer just less than half the number and other integer just more than half the number. If the number is $2n + 1$, the numbers are n and $n + 1$.