ANSWER on Question #79226 – Math – Differential Equations

QUESTION

Find the solution of

$$y' = 2xy^2$$

$$1)\ \ y = \frac{1}{2x} \cdot (1 - y^2)\ \ or\ \ 1)\ \ y = \frac{1}{2x \cdot (1 - y^2)}$$

$$2)\ \ y = \frac{1}{2x} \cdot (1 + y^2)\ \ or\ \ 2)\ \ y = \frac{1}{2x \cdot (1 + y^2)}$$

$$3)\ \ y = \frac{1}{2} \cdot (1 - y^2)\ \ or\ \ 3)\ \ y = \frac{1}{2 \cdot (1 - y^2)}$$

$$4)\ \ y = \frac{1}{2x^2} \cdot (1 - y^2)\ \ or\ \ 4)\ \ y = \frac{1}{2x^2 \cdot (1 - y^2)}$$

SOLUTION

Hint: In the condition, I provided several options, as the formula editor could understand the entry from the question.

Hint: Even without deciding, one can immediately understand that none of the presented options is a solution.

This is clear from the fact that the first-order differential equation presented does not have an initial condition, and therefore there must be an undefined constant in the response.

Hint: I will solve this equation and show what should be the answer.

Now the solution itself.

To solve this problem, we use the Leibniz's notation

$$y' = \frac{dy}{dx}$$

(More information: https://en.wikipedia.org/wiki/Leibniz%27s_notation)

And the method separation of variables.

(More information: https://en.wikipedia.org/wiki/Separation_of_variables)

In our case,

$$y' = 2xy^2 \to \frac{dy}{dx} = 2xy^2 \Big| \times \left(\frac{dx}{y^2}\right) \to \frac{dy}{y^2} = 2xdx \to \int \frac{dy}{y^2} = \int 2xdx \to \left[\frac{1}{y^2} = y^{-2}\right] \to$$

$$\int y^{-2}dy = 2 \cdot \int x^1 dx \to\to \frac{y^{-2+1}}{-2+1} = 2 \cdot \frac{x^{1+1}}{1+1} + C \to \frac{y^{-1}}{-1} = 2 \cdot \frac{x^2}{2} + C \to -\frac{1}{y} = x^2 + C \to$$

$$\boxed{y = \frac{-1}{x^2 + C}}$$

ANSWER:

None of the answers to your choice is appropriate.

The correct answer to this equation is as follows:

$$y' = 2xy^2 \to y = \frac{-1}{x^2 + C}$$