**Question**

$$\frac{dy}{dx} + y = \cos 3x$$

$$y_p = \frac{1}{10}(-3\sin 3x - \cos 3x)$$

$$y_p = \frac{1}{10}(-3\sin 3x + \cos 3x)$$

$$y_p = \frac{1}{10}(3\sin 3x - \cos 3x)$$

$$y_p = \frac{1}{10}(3\sin 3x + \cos 3x)$$

**Solution**

First order non-homogeneous differential equation

$$\frac{dy}{dx} + y = \cos 3x$$

Solve the homogeneous equation

$$\frac{d\tilde{y}}{dx} + \tilde{y} = 0$$

$$\frac{d\tilde{y}}{\tilde{y}} = -dx$$

$$\int \frac{d\tilde{y}}{\tilde{y}} = -\int dx$$

$$\ln|\tilde{y}| = -x + \ln C$$

$$\tilde{y} = Ce^{-x}$$

Let $y_p = A\sin 3x + B\cos 3x$

Then

$$y_p' = 3A\cos 3x - 3B\sin 3x$$

$$y_p' + y_p = \cos 3x$$

$$3A\cos 3x - 3B\sin 3x + A\sin 3x + B\cos 3x = \cos 3x$$

$$\begin{cases} 3A + B = 1 \\ -3B + A = 0 \end{cases} => \begin{cases} 9A + B = 1 \\ A = 3B \end{cases} => \begin{cases} A = \dfrac{3}{10} \\ B = \dfrac{1}{10} \end{cases}$$

$$y_p = \frac{1}{10}(3\sin 3x + \cos 3x)$$

$$y = \tilde{y} + y_p$$

$$y = Ce^{-x} + \frac{1}{10}(3\sin 3x + \cos 3x)$$

**Answer**: the fourth option is correct,      $y_p = \dfrac{1}{10}(3\sin 3x + \cos 3x)$,

$$y = Ce^{-x} + \frac{1}{10}(3\sin 3x + \cos 3x).$$