Answer on Question #79195 – Math – Complex Analysis

Question

Given f(z) = $x^2+y^2-2x+4+i(2xy-2y)$

Express both f(z) and f'(z) in terms of the complex variable z=x+iy

Solution

By condition

$$u(x,y) = \text{Re}\, f(z) = x^2 + y^2 - 2x + 4$$
$$v(x,y) = \text{Im}\, f(z) = 2xy - 2y$$

Then

$$\frac{\partial u}{\partial x} = 2x - 2$$
$$\frac{\partial^2 u}{\partial x^2} = 2$$
$$\frac{\partial u}{\partial y} = 2y$$
$$\frac{\partial^2 u}{\partial y^2} = 2$$

Then

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4 \neq 0$$

But then function $u(x,y)$ is not harmonious and there is no function $f(z)$ with the real part $x^2 + y^2 - 2x + 4$.

Answer: there is no such function.

(perhaps there is a mistake in the condition and the function has the form $x^2 - y^2 - 2x + 4 + i\left(2xy - 2y\right)$, then we can find $f(z)$).

For example,

$$f(z) = x^2 - y^2 - 2x + 4 + i(2xy - 2y)$$

Then

$$u(x,y)=\operatorname{Re} f(z)=x^2-y^2-2x+4$$

$$v(x,y)=\operatorname{Im} f(z)=2xy-2y$$

$$\frac{\partial u}{\partial x}=2x-2$$

$$\frac{\partial v}{\partial x}=2y$$

$$f(z)=f(x+iy)=x^2-y^2-2x+4+i(2xy-2y)=x^2+2ixy-y^2-2x-2iy+4=$$
$$=(x+iy)^2-2(x+iy)+4=z^2-2z+4$$

$$f'(z)=\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}=2x-2+i\cdot 2y=2(x+iy)-2=2z-2$$