Answer on Question #79165 – Math – Calculus

Question

Find the domain of the function defined by $f(x)=\sqrt{9x^3-x^4}$.

Solution

The domain of the function $f(x)=\sqrt{9x^3-x^4}$ is the set of those x for which the expression inside the radical is nonnegative, hence we get the inequality $9x^3-x^4\ge 0$.
To solve the inequality, we use the test-point method.
First solve the equation:

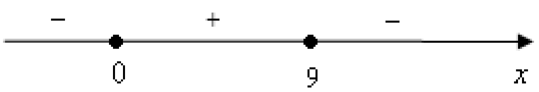
$9x^3-x^4=0$;

$x^3(9-x)=0$;

$x^3=0$ *or* $9-x=0$;

$x=0$ *or* $x=9$;

We use these points to mark intervals on the number line and then we find sign of the factors in each interval.
We choose, for example, the number (-1) on the first interval $(-\infty;0)$. We get $9x^3-x^4=[\,for\ x=-1]=9(-1)^3-(-1)^4=-9-1=-10<0$.
We choose, for example, the number (1) on the second interval $(0;\,9)$. We get $9x^3-x^4=[\,for\ x=1]=9(1)^3-(1)^4=9-1=8>0$.
We choose, for example, the number 10 on the third interval $(9;+\infty)$. We get $9x^3-x^4=[\,for\ x=10]=9(10)^3-(10)^4=9000-10000=-1000<0$.



At the boundary points 0 and 9 the expression $9x^3-x^4$ is zero, so, there is one closed interval $[0;9]$, where $9x^3-x^4\ge 0$.

Therefore, the domain of the function $f(x)=\sqrt{9x^3-x^4}$ is the closed interval $[0;9]$.

Answer: $[0;9]$.