

Answer on Question #79150 – Math – Algebra

Question

1. $5+55+555\dots$ to n terms

Solution

Decompose on multipliers:

$$5 + 55 + 555 + \dots + 55 \dots 5 = 5 \cdot (1 + 11 + 111 + \dots + 11 \dots 1)$$

Multiply and divide this expression by 9:

$$\frac{5}{9} \cdot (9 + 99 + 999 + \dots + 99 \dots 9) = \frac{5}{9} \cdot (10 - 1 + 100 - 1 + 1000 - 1 + \dots + 10^n - 1)$$

We can see a pattern:

$$\begin{aligned} \frac{5}{9} \cdot (10^1 - 1 + 10^2 - 1 + 10^3 - 1 + \dots + 10^n - 1) &= \\ &= \frac{5}{9} \cdot (10^1 + 10^2 + 10^3 + \dots + 10^n - n) \end{aligned}$$

Geometric sequence (progression):

$$10^1, 10^2, 10^3, \dots, 10^n$$

$$a_1 = 10^1$$

$$a_n = 10^n$$

the ratio is $r = 10$

$$Sum = \frac{a_1 \cdot (r^n - 1)}{r - 1}$$

$$Sum = \frac{10 \cdot (10^n - 1)}{9}$$

So

$$\frac{5}{9} * (10^1 + 10^2 + 10^3 + \dots + 10^n - n) = \frac{5}{9} \cdot \left(\frac{10 \cdot (10^n - 1)}{9} - n \right)$$

Answer: $\frac{5}{9} \cdot \left(\frac{10 \cdot (10^n - 1)}{9} - n \right)$.