ANSWER on Question #79107 – Math – Calculus

QUESTION

Let

$$f(x) = x^4 - 2x^2$$

Find the all c (where c is the interception on the x –axis) in the interval $(-2,2)$ such that

$$f'(x) = 0$$

(Hint: use Rolle's theorem)

SOLUTION

1 STEP: We recall Rolle's theorem

If a real-value function f is:

1) continuous on a proper closed interval $[a, b]$;

2) differentiable on the open interval (a, b);

3) $f(a) = f(b)$.

Then there exists at least one c in the open interval (a, b) such that

$$f'(c) = 0$$

(More information: https://en.wikipedia.org/wiki/Rolle%27s_theorem).

2 STEP: We apply Rolle's theorem to a given function

Since the given function $f(x) = x^4 - 2x^2$ is a linear combination of monomials x^4 and $2x^2$ it is continuous and differentiable on any open interval (a, b), in addition to $(-2,2)$. We have only to verify the third condition of the theorem.

$$\begin{cases} f(-2) = (-2)^4 - 2 \cdot (-2)^2 = 16 - 2 \cdot 4 = 16 - 8 = 8 \\ f(2) = 2^4 - 2 \cdot 2^2 = 16 - 2 \cdot 4 = 16 - 8 = 8 \end{cases} \rightarrow \boxed{f(-2) = 8 = f(2)}$$

Conclusion,

$$\textit{All the conditions of Rolle's theorem are satisfied, so we can apply it}$$

$$f(x) = x^4 - 2x^2 \rightarrow f'(x) = (x^4 - 2x^2)' = (x^4)' - (2x^2)' = 4 \cdot x^{4-1} - 2 \cdot 2x^{2-1} = 4x^3 - 4x$$

Then,

$$f'(c) = 0 \to 4c^3 - 4c = 0 \to 4c \cdot (c^2 - 1) = 0 \to \left[\begin{matrix} 4c = 0 \\ c^2 - 1 = 0 \end{matrix}\right. \to \left[\begin{matrix} 4c = 0 \\ c^2 = 1 \end{matrix}\right. \to \boxed{\left[\begin{matrix} c = 0 \\ c = 1 \\ c = -1 \end{matrix}\right.}$$

Conclusion,

$$All\ three\ roots\ c = -1, c = 0, c = 1\ belong\ to\ an\ open\ interval\ (-2,2)$$

3 STEP: We choose the necessary root.

By the condition of the problem, the necessary root must have two properties:

1) $f'(c) = 0$

2) c is the interception on the x −axis.

The second condition means that $f(c) = 0$. Let us check how a previously found root satisfies this condition.

$$c = -1 \to f(-1) = (-1)^4 - 2 \cdot (-1)^2 = 1 - 2 \cdot 1 = 1 - 2 = -1 \neq 0$$

$$c = 0 \to f(0) = 0^4 - 2 \cdot 0^2 = 0 - 2 \cdot 0 = 0 - 0 = 0$$

$$c = 1 \to f(1) = 1^4 - 2 \cdot 1^2 = 1 - 2 \cdot 1 = 1 - 2 = -1 \neq 0$$

Conclusion,

$$\boxed{c = 0\ satisfies\ all\ the\ conditions\ of\ the\ problem}$$

ANSWER: $c = 0$.