

Answer on Question #79086 – Math – Calculus

Question

Differentiate $\tan^{-1}\left\{\frac{\sin x - \cos x}{\sin x + \cos x}\right\}$ with respect to $x/2$.

Solution

$$\begin{aligned}\text{Let, } a &= \tan^{-1}\left[\frac{(\sin x - \cos x)}{(\sin x + \cos x)}\right] \\ &= \tan^{-1}\left[\frac{(\tan x - 1)}{(\tan x + 1)}\right] \quad \left[\text{As, } \frac{(\sin x - \cos x)}{(\sin x + \cos x)} = \frac{\frac{(\sin x - \cos x)}{\cos x}}{\frac{(\sin x + \cos x)}{\cos x}} = \frac{(\tan x - 1)}{(\tan x + 1)}\right] \\ &= \tan^{-1}\left[\tan\left(x - \frac{\pi}{4}\right)\right] \quad \left[\text{As, } \tan \frac{\pi}{4} = 1 \text{ and } \tan(a-b) = \frac{(\tan a - \tan b)}{(\tan a \cdot \tan b + 1)}\right] \\ &= \left(x - \frac{\pi}{4}\right)\end{aligned}$$

Now differentiate a with respect to x we get, $\frac{da}{dx} = 1$

Again say, $b = \frac{x}{2}$

Now differentiate b with respect to x we get, $\frac{db}{dx} = \frac{1}{2}$

$$\text{So, } \frac{da}{db} = \frac{\frac{da}{dx}}{\frac{db}{dx}} = \frac{1}{\frac{1}{2}} = 2.$$

Answer: $\frac{da}{db} = 2$.