Answer on Question #79085 – Math – Calculus

Question

If $f(x)=\frac{4x^2-7x-2}{x-2}$, $x\neq 0$, find $\delta>0$ such that $|f(x)-9|<\frac{1}{100}$ for $0<|x-2|<\delta$.

Hence show $\lim\limits_{x\to 2} f(x)=9$

Solution

We have that

$$|f(x)-9|=\left|\frac{4x^2-7x-2}{x-2}-9\right|=\left|\frac{(x-2)(4x+1)}{x-2}-9\right|=$$

$$=|4x+1-9|=4|x-2|<\frac{1}{100}$$

Let $\delta=\frac{1}{400}$. Then for $0<|x-2|<\frac{1}{400}$

$$|f(x)-9|=\left|\frac{4x^2-7x-2}{x-2}-9\right|=\left|\frac{(x-2)(4x+1)}{x-2}-9\right|=$$

$$=|4x+1-9|=4|x-2|<4\left(\frac{1}{400}\right)=\frac{1}{100}$$

$\delta=\frac{1}{400}$.

Let $\delta=\frac{1}{4}\varepsilon, \varepsilon>0$. Then for $0<|x-2|<\delta$

$$|f(x)-9|=\left|\frac{4x^2-7x-2}{x-2}-9\right|=\left|\frac{(x-2)(4x+1)}{x-2}-9\right|=$$

$$=|4x+1-9|=4|x-2|<4\delta=\varepsilon$$

Therefore, $\forall\varepsilon>0, \exists\delta(\varepsilon)>0$ such that $|f(x)-9|<\varepsilon$, with $0<|x-2|<\delta$

$$\lim_{x\to 2} f(x)=\lim_{x\to 2}\left(\frac{4x^2-7x-2}{x-2}\right)=9$$